(7 pages)

Reg. No. : ....

Code No. : 20069 E **Sub. Code : SMMA 63**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The maximum degree of a point in a graph with p points is ————

(a) p (b) $p-1$

(c) q (d) p^2

2. If G is a (p,q) graph, then ————

(a) $q \le \binom{p}{2}$ (b) $q = \binom{p}{2}$

(c) $q \ge \binom{p}{2}$ (d) $q = p-1$

3. Which of the following is a graphic sequence?

(a) (1, 1, 1) (b) (2, 2, 1)

(c) (2, 1, 1) (d) (1, 0, 0)

4. The connectivity of the complete graph K_p is ————

(a) p (b) 0

(c) 1 (d) $p-1$

5. Which of the following is an Eulerian graph?

(a) K_6 (b) K_7

(c) $K_{3,3}$ (d) $K_{2,5}$

6. Every Hamiltonian graph is ———— connected.

(a) 2 (b) p

(c) $p-1$ (d) q

7. If G is a maximal planar (p,q) graph, then ————

(a) $q \le 2p-4$ (b) $q \le 3p-6$

(c) $q \ge 3p-6$ (d) $q = 3p-6$

8. $\chi(K_{2,10}) =$

(a) 10 (b) 20

(c) 2 (d) 12

9. If G is a (p,q) graph, then the coefficient of λ^{p-1} in $f(G,\lambda)$ is ————

(a) 0 (b) q

(c) $-q$ (d) p

10. If a complete digraph has n vertices, then it has ———— arcs.

(a) $n(n-1)$ (b) $\frac{n(n-1)}{2}$

(c) $n-1$ (d) $n(n+1)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Prove : $\alpha' + \beta' = p$.

Or

(b) Prove that any self complementary graph has $4n$ or $4n+1$ points.

12. (a) Verify whether the partition (4, 4, 4, 2, 2, 2) is graphical. If it is graphical, draw the corresponding graph.

Or

(b) Prove : A line x of a connected graph G is a bridge if and only if x is not on any cycle of G.

13. (a) If G is a graph with $p \ge 3$ and $\delta \ge \frac{p}{2}$, then show that G is Hamiltonian.

Or

(b) Prove that every tree has a center consisting of either one point or two adjacent points.

[P.T.O.]

14. (a) State prove that Euler's theorem on a connected plane graph.

Or

(b) Show that every uniquely n-colourable graph is $(n-1)$-connected.

15. (a) Prove that $\lambda^4 - 3\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

Or

(b) Define :

(i) Strongly connected digraph

(ii) Unilaterally connected digraph.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Show that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

Or

(b) Let G_1 be a (p_1,q_1) graph and G_2 be a (p_2,q_2) graph. Then prove :

(i) $G_1 \times G_2$ is a (p_1p_2, q_1p_2, q_2p_1) graph

(ii) $G_1[G_2]$ is a $(p_1p_2, p_1q_2, p_2^2q_1)$ graph.

17. (a) State and prove a necessary and sufficient condition for a partition $P = (d_1, d_2, ..., d_p)$ of an even number into parts with $p-1 \geq d_1 \geq d_2 \geq \cdots d_p$ to be graphical.

Or

(b) Show that a graph G with at least two points is bipartite if and only if all its cycles are of even length.

18. (a) Prove that $c(G)$ is well define.

Or

(b) Let G be a (p,q)- graph. Prove that the following are equivalent :

(i) G is a tree

(ii) Every two points of G are joined by a unique path

(iii) G is connected and $p = q+1$

(iv) G is acyclic and $p = q+1$.

19. (a) Prove : $\chi'(K_n) = n$, if $n \neq 1$ is odd

$= n-1$, if n is even.

Or

(b) Show that K_5 and $K_{3,3}$ are non planar graphs.

20. (a) If G is a tree with $n \geq 2$ points, then show that $f(G,\lambda) = \lambda(\lambda-1)^{n-1}$.

Or

(b) Prove that the edges of connected graph G can be oriented so that the regulating digraph is strongly connected if and only if every edge of G is contained in at least one cycle.

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